Energy Conversion – Exam I

School of Engineering – Dep. of Industrial & Mechanical Eng.

Name: Solutio Key
Date: Tuesday, April 9th 2013; 06:00 PM
Location: ENG Attic
Instructor: Dr. Wassim Habchi
Notes: No documents allowed
Value: 25% of Total Grade
Time: 2 hours

[___ / 100]

Problem I (20 points)

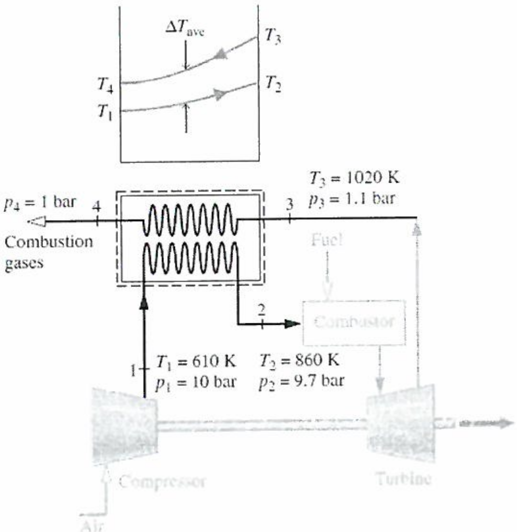
Compressed air enters a counter-flow heat exchanger operating at steady state at 610 K, 10 bar and exits at 860K, 9.7 bar (1bar=10^5Pa). Hot combustion gas enters as a separate stream at 1020 K, 1.1 bar and exits at 1 bar. Each stream has a mass flow rate of 90 kg/s. Heat transfer between the outer surface of the heat exchanger and the surroundings can be ignored. Kinetic and potential energy effects are negligible. The temperature and pressure of the dead state are T_0=300K and P_0=1bar. Assuming the combustion gas stream has the properties of air, and using the ideal gas model for both streams, determine for the heat exchanger:

a) The exit temperature T_4 of the combustion gas, in K. (**5 points**)

b) The net change in the flow exergy rate from inlet to exit of each stream, in MW. (**5 points**)

c) The rate of exergy destruction, in MW. (**5 points**)

d) The second-law efficiency of the heat exchanger. (**5 points**)



Solution:

a) Energy balance on the heat exchanger:

$h_3 - h_4 = h_2 - h_1 \longrightarrow \boxed{h_4 = h_3 - h_2 + h_1}$

A-17: $h_1 = 617.53$ KJ/Kg, $h_2 = 888.27$ KJ/Kg, $h_3 = 1068.89$ KJ/Kg

$\Rightarrow h_4 = 1068.89 - 888.27 + 617.53 = 798.15 \ KJ/Kg.$

A-17 $\longrightarrow$ $\boxed{T_4 = 778.28K}$

b) Hot stream: $\psi_4 - \psi_3 = h_4 - h_3 - T_0(s_4 - s_3)$

Cold stream: $\psi_2 - \psi_1 = h_2 - h_1 - T_0(s_2 - s_1)$

but for a ideal gas:

- $s_4 - s_3 = s_4^\circ - s_3^\circ - R \ln\left(\frac{P_4}{P_3}\right) = 2.6877 - 2.99034 - 0.287 \ln\left(\frac{1}{1.1}\right)$

$= -0.275286 \ KJ/Kg.K$

- $s_2 - s_1 = s_2^\circ - s_1^\circ - R \ln\left(\frac{P_2}{P_1}\right) = 2.79783 - 2.42644 - 0.287 \ln\left(\frac{9.7}{10}\right)$

$= 0.3801318 \ KJ/Kg.K$

$\Rightarrow$ Hot Stream: $\psi_4 - \psi_3 = (798.15 - 1068.89) - 300(-0.275286) = -188.1542 \ KJ/Kg$

Cold Stream: $\psi_2 - \psi_1 = 888.27 - 617.53 - 300(0.3801318) = 156.7 \ KJ/Kg$

$\Rightarrow$ Hot Stream: $\Delta \dot{X}_h = \dot{m}(\psi_4 - \psi_3) = 90 \times (-188.1542) = -16933.88 KW = \boxed{-16.9MW}$

Cold Stream: $\Delta \dot{X}_c = \dot{m}(\psi_2 - \psi_1) = 90 \times 156.7 = 14103 \ KW = \boxed{14.1MW}$

c) Exergy balance:

$\dot{m}(\psi_1 - \psi_2) + \dot{m}(\psi_3 - \psi_4) - \dot{X}_{dest} = 0$

$\rightarrow \dot{X}_{dest} = -\Delta\dot{X}_c - \Delta\dot{X}_h = -14.1 + 16.9 = \boxed{2.8 \ MW}$

d) $\eta_{II} = \frac{\Delta\dot{X}_c}{-\Delta\dot{X}_h} = \frac{14.1}{16.9} = 0.8343 = \boxed{83.43\%}$

or $\eta_{II} = 1 - \frac{\dot{X}_{dest}}{-\Delta\dot{X}_h} = 1 - \frac{2.8}{16.9} = 0.8343 = \boxed{83.43\%}$

Problem II (25 points)

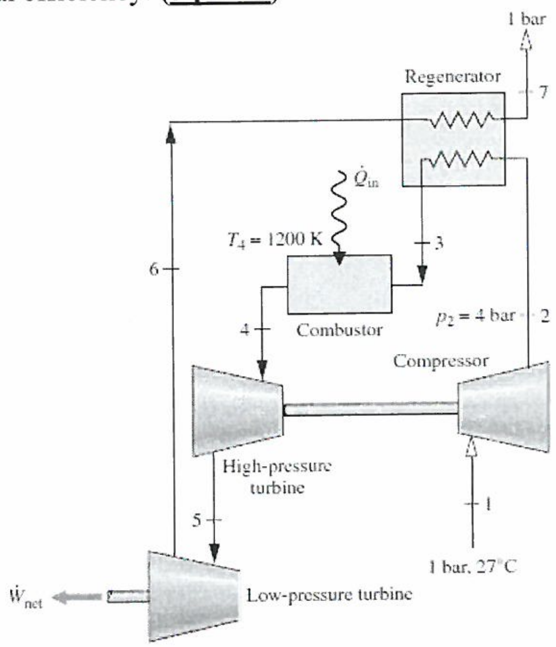
Consider a regenerative gas turbine power plant as shown below. Air enters the compressor at 1 bar, 27°C with a mass flow rate of 0.562 kg/s and is compressed to 4 bars (1bar=10^5Pa). The isentropic efficiency of the compressor is 80%, and the regenerator effectiveness is 90%. All the power developed by the high-pressure turbine is used to run the compressor. The low-pressure turbine provides the net power output. ~~Each~~ The turbine has an overall isentropic efficiency of 87% and the temperature at the inlet to the high-pressure turbine is 1200K.

a) Sketch a *T-s* diagram of the corresponding cycle. (**5 points**)

b) Determine the temperature of the air at states 2, 3, 5, 6, and 7, in K. (**~~10~~ 5 points**)

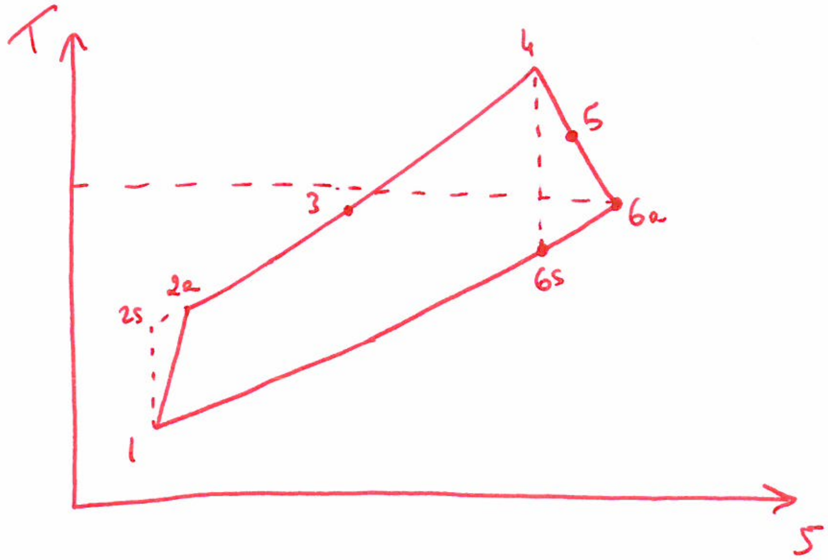
b) Determine the net power output, in kW. (**5 points**)

c) Determine the thermal efficiency. (**5 points**)



Solution:

a)



b) $\frac{h_{2s}-h_1}{h_{2a}-h_1} = 0.8 \Rightarrow h_{2a} = \left(\frac{h_{2s}-h_1}{0.8}\right) + h_1$

A-17: $h_1 = 300.19$ kJ/kg $\quad P_{r_1} = 1.386$

but $\frac{P_{r_2}}{P_{r_1}} = \frac{P_2}{P_1} = 4 \Rightarrow P_{r_2} = 4P_{r_1} = 4\times1.386 = 5.544$

$\rightarrow h_{2s} = 446.49\ KJ/Kg$ but $h_{2a} = \dfrac{446.49 - 300.19}{0.8} + 300.19$

$h_{2a} = 483.065\ KJ/Kg$

$\Rightarrow \boxed{T_{2a} = 480.56\ K}$

Now $\dot{W}_{turb\,I} = \dot{W}_{comp} \Rightarrow h_4 - h_5 = h_2 - h_1$

$\Rightarrow h_5 = h_4 - h_2 + h_1$ but $h_4 = 1277.79\ KJ/Kg$ and $P_{r_4} = 238$

$\Rightarrow h_5 = 1277.79 - 483.065 + 300.19$

$h_5 = 1094.9\ KJ/Kg \Rightarrow \boxed{T_5 = 1042.65\ K}$

$\dfrac{P_{r_6}}{P_{r_4}} = \dfrac{P_6}{P_4} = \dfrac{1}{4} \Rightarrow P_{r_6} = \dfrac{P_{r_4}}{4} = \dfrac{238}{4} = 59.5$

$\Rightarrow h_{6s} = 873.76\ KJ/Kg$

but $\dfrac{h_{6a} - h_4}{h_{6s} - h_4} = 0.87 \rightarrow h_{6a} = 0.87(873.76 - 1277.79) + 1277.79$

$= 926.284\ KJ/Kg$

$\Rightarrow \boxed{T_{6a} = 894.06\ K}$

For the regenerator: $\varepsilon = \dfrac{h_3 - h_2}{h_6 - h_2} = 0.9 \Rightarrow h_3 = 0.9(h_6 - h_2) + h_2$

$= 0.9(926.284 - 483.065) + 483.065$

$= 881.96\ KJ/Kg$

$\Rightarrow \boxed{T_3 = 854.3\ K}$

En. balance on regenerator: $h_6 - h_7 = h_3 - h_2$

$\Rightarrow h_7 = h_6 - h_3 + h_2 = 926.284 - 881.96 + 483.065 = 527.39\ KJ/Kg$

$\Rightarrow \boxed{T_7 = 523.63\ K}$

-) $\dot{W}_{net} = \dot{m}(h_5 - h_6) = 0.562(1094.9 - 926.284) = \boxed{94.76\ KW}$

c) $\eta_{th} = \dfrac{\dot{W}_{net}}{\dot{Q}_{in}} = \dfrac{\dot{W}_{net}}{\dot{m}(h_4 - h_3)} = \dfrac{94.76}{0.562(1277.79 - 881.96)} = \boxed{0.426}$

Problem III (25 points)

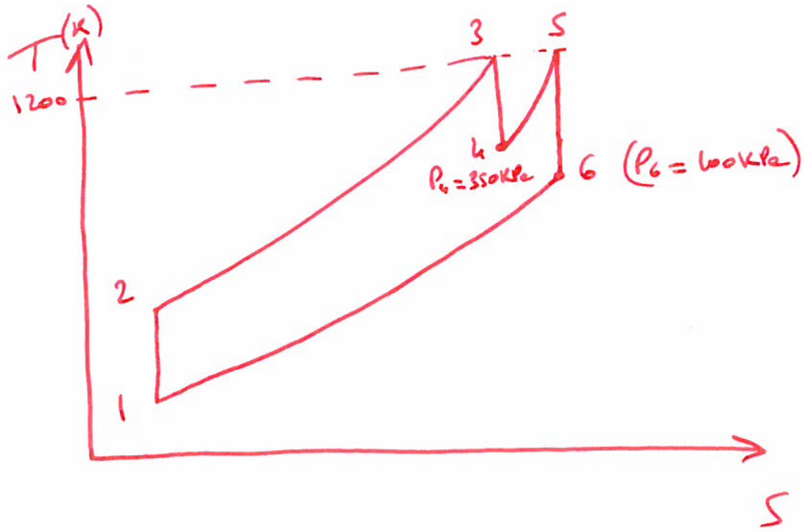
Air enters the turbine of a gas turbine at 1200 kPa, 1200 K, and expands to 100 kPa in two stages. Between the stages, the air is reheated at a constant pressure of 350 kPa to 1200 K. The expansion through each turbine stage is isentropic. Determine, in **kJ per kg of air** flowing through the cycle:

a) The work developed by each stage. **(10 points)**

b) The heat transfer for the reheat process. **(5 points)**

c) The increase in net work output as compared to a single stage of expansion with no reheat. (~~10~~ 5 **points**)

Solution:



a) $w_I = h_3 - h_4$ @ ③: $T_3 = 1200$ K → $h_3 = 1277.79$ kJ/kg, $P_{r_3} = 238$

but $\frac{P_{r_4}}{P_{r_3}} = \frac{P_4}{P_3} = \frac{350}{1200}$ → $P_{r_4} = \frac{350}{1200} \times 238 = 69.42$

→ $h_4 = 912.12$ kJ/kg

⇒ $w_I = 1277.79 - 912.12 = 365.67$ kJ/kg

And $w_{II} = h_5 - h_6$ but $h_5 = h_3 = 1277.79$ kJ/kg & $P_{r_5} = 238$

and $\frac{P_{r_6}}{P_{r_5}} = \frac{P_6}{P_5} = \frac{100}{350}$ ⇒ $P_{r_6} = \frac{100}{350} \times 238 = 68$

→ $h_6 = 906.85$ kJ/kg

⇒ $w_{II} = 1277.79 - 906.85 = 370.94$ kJ/kg

b) $q_{reheat} = h_5 - h_4 = 1277.79 - 912.12 = \boxed{365.67\ KJ/Kg}$

c) Single-Stage expansion (3-4):

$$\frac{P_{r_4}}{P_{r_3}} = \frac{P_4}{P_3} = \frac{100}{1200} \rightarrow P_{r_4} = \frac{100}{1200} \times 238 = 19.83$$

$$\rightarrow \boxed{h_4 = 638.6\ KJ/Kg}$$

And $w = h_3 - h_4 = 1277.79 - 638.6 = \boxed{639.19\ KJ/Kg}$

Double-stage expansion

$$w = w_I + w_{II} = 365.67 + 370.94 = \boxed{736.61\ KJ/Kg}$$

Since w_{comp} is not affected:

$$\Delta w_{net} = 736.61 - 639.19 = \boxed{97.42\ KJ/Kg}$$

Problem IV (30 points)

The conditions at the beginning of compression in an air-standard Diesel cycle are P_1= 200 kPa, T_1=380 K. The compression ratio is 20 and the heat addition per unit mass q_{in}=900 kJ/kg. Using cold-air standard assumptions:

a) Sketch a P-v diagram of the corresponding cycle. (**5 points**)

b) Determine the maximum temperature of the cycle, in K. (**5 points**)

c) Determine the cutoff ratio. (**5 points**)

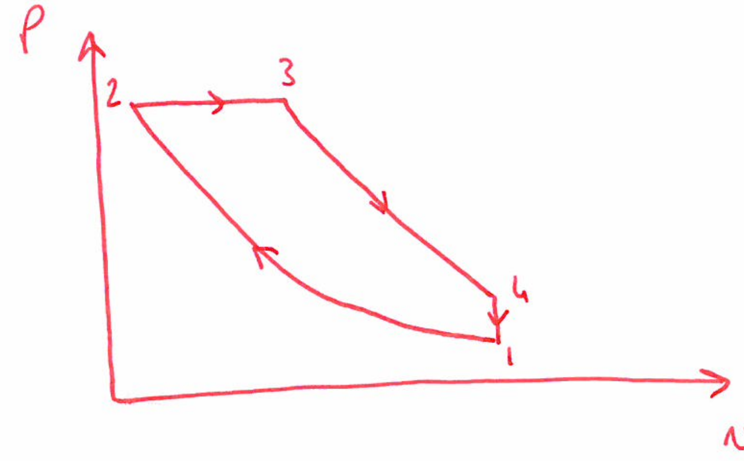
d) Determine the net work output per unit mass of air, in kJ/kg. (**5 points**)

e) Determine the thermal efficiency of the cycle. (**5 points**)

f) Determine the mean effective pressure, in kPa. (**5 points**)

Solution:

a)



$P_1 = 200$ kPa
$T_1 = 380$ K
$r = 20$
$q_{in} = 900$ kJ/kg

b) A-2: $R = 0.287$ kJ/kg.K $c_p = 1.005$ kJ/kg.K
$c_v = 0.718$ kJ/kg.K $k = 1.4$

• 1→2 isentropic: $P_1 v_1^k = P_2 v_2^k \Rightarrow P_2 = P_1\left(\frac{v_1}{v_2}\right)^k$

$\Rightarrow P_2 = 200 \times 20^{1.4} = 13257.82$ kPa

and $\frac{P_2 v_2}{T_2} = \frac{P_1 v_1}{T_1} \Rightarrow T_2 = \frac{P_2 v_2}{P_1 v_1} \times T_1 = \frac{13257.82}{200} \times \frac{1}{20} \times 380$

$\Rightarrow$ $\boxed{T_2 = 1259.5 \text{ K}}$

• 2-3 constant-pressure heat addition:

$q_{in} - \int_2^3 p\,dv = u_3 - u_2$

$q_{in} - P(v_3 - v_2) = u_3 - u_2$

$\Rightarrow q_{in} = (u_3 + P_3 v_3) - (u_2 + P_2 v_2) = h_3 - h_2 = c_p(T_3 - T_2)$

$\Rightarrow T_3 = \frac{q_{in}}{c_p} + T_2 = \frac{900}{1.005} + 1253.5 = \boxed{2155K}$

c) $r_c = \frac{v_3}{v_2} = \frac{T_3}{T_2} = \frac{2155}{1253.5} = \boxed{1.71}$

d) $w_{net,out} = q_{net,in} = q_{in} - q_{out} = q_{in} - c_v(T_4 - T_1)$

but 3-4 is isentropic: $T_4 v_4^{k-1} = T_3 v_3^{k-1}$

$\Rightarrow T_4 = T_3 \left(\frac{v_3}{v_4}\right)^{k-1} = T_3 \left(\frac{r_c v_2}{r v_2}\right)^{k-1}$

$\Rightarrow T_4 = T_3 \left(\frac{r_c}{r}\right)^{k-1} = 2155 \times \left(\frac{1.71}{20}\right)^{0.4} = \boxed{805.8K}$

$\Rightarrow w_{net,out} = 900 - 0.718(805.8 - 380) = \boxed{594.28\ kJ/kg}$

e) $\eta_{th} = \frac{w_{net,out}}{q_{in}} = \frac{594.28}{900} = 0.6603 = \boxed{66.03\%}$

f) $MEP = \frac{w_{net,out}}{v_1 - v_2} = \frac{w_{net,out}}{v_1 - \frac{v_1}{r}} = \frac{w_{net,out}}{v_1\left(1 - \frac{1}{r}\right)} = \frac{w_{net,out}}{\frac{RT_1}{P_1}\left(1 - \frac{1}{r}\right)}$

$MEP = \frac{594.28}{\frac{0.287 \times 380}{200} \times \left(1 - \frac{1}{20}\right)} = \boxed{1147.18\ kPa}$

TABLE A–2

Ideal-gas specific heats of various common gases

(*a*) At 300 K

Gas	Formula	Gas constant, *R* kJ/kg · K	c_p kJ/kg · K	c_v kJ/kg · K	*k*
Air	—	0.2870	1.005	0.718	1.400
Argon	Ar	0.2081	0.5203	0.3122	1.667
Butane	C_4H_{10}	0.1433	1.7164	1.5734	1.091
Carbon dioxide	CO_2	0.1889	0.846	0.657	1.289
Carbon monoxide	CO	0.2968	1.040	0.744	1.400
Ethane	C_2H_6	0.2765	1.7662	1.4897	1.186
Ethylene	C_2H_4	0.2964	1.5482	1.2518	1.237
Helium	He	2.0769	5.1926	3.1156	1.667
Hydrogen	H_2	4.1240	14.307	10.183	1.405
Methane	CH_4	0.5182	2.2537	1.7354	1.299
Neon	Ne	0.4119	1.0299	0.6179	1.667
Nitrogen	N_2	0.2968	1.039	0.743	1.400
Octane	C_8H_{18}	0.0729	1.7113	1.6385	1.044
Oxygen	O_2	0.2598	0.918	0.658	1.395
Propane	C_3H_8	0.1885	1.6794	1.4909	1.126
Steam	H_2O	0.4615	1.8723	1.4108	1.327

Note: The unit kJ/kg · K is equivalent to kJ/kg · °C.

Source: Chemical and Process Thermodynamics 3/E by Kyle, B. G., © 2000. Adapted by permission of Pearson Education, Inc., Upper Saddle River, NJ.

TABLE A–17

Ideal-gas properties of air

T K	h kJ/kg	P_r	u kJ/kg	v_r	s° kJ/kg · K
200	199.97	0.3363	142.56	1707.0	1.29559
210	209.97	0.3987	149.69	1512.0	1.34444
220	219.97	0.4690	156.82	1346.0	1.39105
230	230.02	0.5477	164.00	1205.0	1.43557
240	240.02	0.6355	171.13	1084.0	1.47824
250	250.05	0.7329	178.28	979.0	1.51917
260	260.09	0.8405	185.45	887.8	1.55848
270	270.11	0.9590	192.60	808.0	1.59634
280	280.13	1.0889	199.75	738.0	1.63279
285	285.14	1.1584	203.33	706.1	1.65055
290	290.16	1.2311	206.91	676.1	1.66802
295	295.17	1.3068	210.49	647.9	1.68515
298	298.18	1.3543	212.64	631.9	1.69528
300	300.19	1.3860	214.07	621.2	1.70203
305	305.22	1.4686	217.67	596.0	1.71865
310	310.24	1.5546	221.25	572.3	1.73498
315	315.27	1.6442	224.85	549.8	1.75106
320	320.29	1.7375	228.42	528.6	1.76690
325	325.31	1.8345	232.02	508.4	1.78249
330	330.34	1.9352	235.61	489.4	1.79783
340	340.42	2.149	242.82	454.1	1.82790
350	350.49	2.379	250.02	422.2	1.85708
360	360.58	2.626	257.24	393.4	1.88543
370	370.67	2.892	264.46	367.2	1.91313
380	380.77	3.176	271.69	343.4	1.94001
390	390.88	3.481	278.93	321.5	1.96633
400	400.98	3.806	286.16	301.6	1.99194
410	411.12	4.153	293.43	283.3	2.01699
420	421.26	4.522	300.69	266.6	2.04142
430	431.43	4.915	307.99	251.1	2.06533
440	441.61	5.332	315.30	236.8	2.08870
450	451.80	5.775	322.62	223.6	2.11161
460	462.02	6.245	329.97	211.4	2.13407
470	472.24	6.742	337.32	200.1	2.15604
480	482.49	7.268	344.70	189.5	2.17760
490	492.74	7.824	352.08	179.7	2.19876
500	503.02	8.411	359.49	170.6	2.21952
510	513.32	9.031	366.92	162.1	2.23993
520	523.63	9.684	374.36	154.1	2.25997
530	533.98	10.37	381.84	146.7	2.27967
540	544.35	11.10	389.34	139.7	2.29906
550	555.74	11.86	396.86	133.1	2.31809
560	565.17	12.66	404.42	127.0	2.33685
570	575.59	13.50	411.97	121.2	2.35531
580	586.04	14.38	419.55	115.7	2.37348
590	596.52	15.31	427.15	110.6	2.39140
600	607.02	16.28	434.78	105.8	2.40902
610	617.53	17.30	442.42	101.2	2.42644
620	628.07	18.36	450.09	96.92	2.44356
630	638.63	19.84	457.78	92.84	2.46048
640	649.22	20.64	465.50	88.99	2.47716
650	659.84	21.86	473.25	85.34	2.49364
660	670.47	23.13	481.01	81.89	2.50985
670	681.14	24.46	488.81	78.61	2.52589
680	691.82	25.85	496.62	75.50	2.54175
690	702.52	27.29	504.45	72.56	2.55731
700	713.27	28.80	512.33	69.76	2.57277
710	724.04	30.38	520.23	67.07	2.58810
720	734.82	32.02	528.14	64.53	2.60319
730	745.62	33.72	536.07	62.13	2.61803
740	756.44	35.50	544.02	59.82	2.63280
750	767.29	37.35	551.99	57.63	2.64737
760	778.18	39.27	560.01	55.54	2.66176
780	800.03	43.35	576.12	51.64	2.69013
800	821.95	47.75	592.30	48.08	2.71787
820	843.98	52.59	608.59	44.84	2.74504
840	866.08	57.60	624.95	41.85	2.77170
860	888.27	63.09	641.40	39.12	2.79783
880	910.56	68.98	657.95	36.61	2.82344
900	932.93	75.29	674.58	34.31	2.84856
920	955.38	82.05	691.28	32.18	2.87324
940	977.92	89.28	708.08	30.22	2.89748
960	1000.55	97.00	725.02	28.40	2.92128
980	1023.25	105.2	741.98	26.73	2.94468
1000	1046.04	114.0	758.94	25.17	2.96770
1020	1068.89	123.4	776.10	23.72	2.99034
1040	1091.85	133.3	793.36	23.29	3.01260
1060	1114.86	143.9	810.62	21.14	3.03449
1080	1137.89	155.2	827.88	19.98	3.05608
1100	1161.07	167.1	845.33	18.896	3.07732
1120	1184.28	179.7	862.79	17.886	3.09825
1140	1207.57	193.1	880.35	16.946	3.11883
1160	1230.92	207.2	897.91	16.064	3.13916
1180	1254.34	222.2	915.57	15.241	3.15916
1200	1277.79	238.0	933.33	14.470	3.17888
1220	1301.31	254.7	951.09	13.747	3.19834
1240	1324.93	272.3	968.95	13.069	3.21751